

Time After Time: Notes on Delays In Spiking Neural P Systems

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Abstract. Spiking Neural P systems, SNP systems for short, are biologically inspired computing devices based on how neurons perform computations. SNP systems use only one type of symbol, the spike, in the computations. Information is encoded in the time differences of spikes or the multiplicity of spikes produced at certain times. SNP systems with delays (associated with rules) and those without delays are two of several Turing complete SNP system variants in literature. In this work we investigate how restricted forms of SNP systems with delays can be simulated by SNP systems without delays. We show the simulations for the following spike routing constructs: sequential, iteration, join, and split.

Keywords: Membrane Computing, Spiking Neural P systems, delays, routing, simulations

1 Introduction

Membrane computing¹ abstracts computational ideas from the way biological cells perform information processing. The models used in Membrane computing are known as P systems. Spiking Neural P systems, or SNP systems for short, are a class of P systems that incorporate the neural-like P systems which only use indistinguishable electric signals or spikes. SNP systems were introduced in [8] and were proven in [8] and [4] to be Turing-complete. Because SNP systems only use one kind of spike symbol a , the output of the computation done by the system is based on the time interval between the first spike and the second spike of a designated output neuron. This model represents the fact that spikes in a biological neural system are almost identical from an electrical point of view. The time intervals between these signal spikes are crucial to the computations performed by neurons.

As with the more common results in Membrane computing, SNP systems have been used to efficiently solve hard problems such as the SAT problem

¹ a relatively new field of Unconventional and Natural computing introduced in 1998 by Gheorghe Păun, see e.g. [10]

using exponential workspace [9]. Other numerous results also focus on using SNP systems as acceptors, generators, and transducers as in [7] and [1]. SNP systems were also used in order to perform arithmetic operations where the operands are encoded in their binary formats [5]. It has been proven in [6] that delays in applying rules in the neurons of an SNP system are not required for SNP systems to be Turing complete. However SNP systems with delays might still prove to be useful for modeling purposes. In [13] SNP systems without delays were represented as matrices and the computations starting from an initial configuration can be performed using linear algebra operations. By simulating SNP systems with delays using SNP systems without delays, the work in [13] can be used to further study SNP systems.²

In this work we investigate how SNP systems with delays can be simulated by SNP systems without delays. In particular we focus on a special type of SNP systems that consist of a source neuron and a sink neuron. The initial configuration consists of a spike in the source neuron only and a final configuration where only the sink neuron has a spike. The spike from the source neuron to the sink neuron is routed through other intermediary neurons using four routing primitives: *sequential*, *iteration*, *joins*, and *splits*.

The contents of this paper are organized as follows: Section 2 provides definitions and notations that will be used in this work. Section 3 presents our results. We end this paper in Section 4 with some final notes, conjectures and open problems.

2 Preliminaries

It is assumed that the readers are familiar with the basics of Membrane Computing³ and formal language theory. We only briefly mention notions and notations which will be useful throughout the paper. Let V be an alphabet, V^* is the free monoid over V with respect to concatenation and the identity element λ (the empty string). The set of all non-empty strings over V is denoted as V^+ so $V^+ = V^* - \{\lambda\}$. We call V a *singleton* if $V = \{a\}$ and simply write a^* and a^+ instead of $\{a^*\}$ and $\{a^+\}$. The length of a string $w \in V^*$ is denoted by $|w|$. If a is a symbol in V , $a^0 = \lambda$. A language $L \subseteq V^*$ is regular if there is a regular expression E over V such that $L(E) = L$. A regular expression over an alphabet V is constructed starting from λ and the symbols of V using the operations union, concatenation, and $+$, using parentheses when necessary to specify the order of operations. Specifically, (i) λ and each $a \in V$ are regular expressions, (ii) if E_1 and E_2 are regular expressions over V then $(E_1 \cup E_2)$, $E_1 E_2$, and E_1^+ are regular expressions over V , and (iii) nothing else is a regular expression over V . With each expression E we associate a language $L(E)$ defined in the following way: (i) $L(\lambda) = \{\lambda\}$ and $L(a) = \{a\}$ for all $a \in V$, (ii)

² A massively parallel SNP system simulator in [2,3] is based on the matrix representation in order to perform simulations.

³ a good introduction is [10] with recent results and information in the P systems webpage at <http://ppage.psyste.ms.eu/> and a recent handbook in [12]

$L(E_1 \cup E_2) = L(E_1) \cup L(E_2)$, $L(E_1 E_2) = L(E_1)L(E_2)$, and $L(E_1^+) = L(E_1)^+$, for all regular expressions E_1, E_2 over V . Unnecessary parentheses are omitted when writing regular expressions, and $E^+ \cup \{\lambda\}$ is written as E^* .

We define an SNP system of a finite degree $m \geq 1$ as follows:

$$\Pi = (O, \sigma_1, \dots, \sigma_m, syn),$$

where:

1. $O = \{a\}$ is the singleton alphabet (a is called *spike*).
2. $\sigma_1, \dots, \sigma_m$ are neurons of the form $\sigma_i = (\alpha_i, R_i)$, $1 \leq i \leq m$, where:
 - (a) $\alpha_i \geq 0$ is an integer representing the initial number of spikes in σ_i
 - (b) R_i is a finite set of rules of the general form

$$E/a^c \rightarrow a^b; d$$

where E is a regular expression over O , $c \geq 1$, $b \geq 0$, with $c \geq b$.

3. $syn \subseteq \{1, 2, \dots, m\} \times \{1, 2, \dots, m\}$, $(i, i) \notin syn$ for $1 \leq i \leq m$, are synapses between neurons.

A *spiking rule* is where $b \geq 1$. A *forgetting rule* is a rule where $b = 0$ is written as $E/a^c \rightarrow \lambda$. If $L(E) = \{a^c\}$ then spiking and forgetting rules are simply written as $a^c \rightarrow a^b$ and $a^c \rightarrow \lambda$, respectively. Applications of rules are as follows: if neuron σ_i contains k spikes, $a^k \in L(E)$ and $k \geq c$, then the rule $E/a^c \rightarrow a^b \in R_i$ is enabled and the rule can be fired or applied. If $b \geq 1$, the application of this rule removes c spikes from σ_i , so that only $k - c$ spikes remain in σ_i . The neuron fires b number of spikes to every σ_j such that $(i, j) \in syn$. If the delay $d = 0$, the b number of spikes are sent immediately i.e. in the same time step as the application of the rule. If $d \geq 1$ and the rule with delay was applied at time t , then the spikes are sent at time $t + d$. From time t to $t + d - 1$ the neuron is said to be *closed*⁴ and cannot receive spikes. Any spikes sent to the neuron when the neuron is closed are *lost* or removed from the system. At time $t + d$ the neuron becomes *open* and can then receive spikes again. The neuron can then apply another rule at time $t + d + 1$. If $b = 0$ then no spikes are produced. SNP systems assume a global clock, so the application of rules and the sending of spikes by neurons are all synchronized.

A configuration of the system can be denoted as $\langle n_1/t_1, \dots, n_m/t_m \rangle$, where each element of the vector is the configuration of a neuron σ_i , with n_i spikes and is open after $t_i \geq 0$ steps. At time step 0, since no rules, with or without delay, are yet to be applied, $t_i = 0, \forall i \in \{1, \dots, m\}$. t_i increases when σ_i applies a rule with delay.

To illustrate, let us have an snp system with three neurons, σ_1 , σ_2 , and σ_3 with synapses $(1, 2)$, $(2, 3)$. Only σ_1 has some number of spikes, say x . σ_1 has a rule $a^+/a \rightarrow a$; σ_2 has a rule $a \rightarrow a$, and σ_3 is a sink neuron. At a time step k where no rules are yet to be applied, the configuration of this system would be $\langle x/0, 0/0, 0/0 \rangle$. At time step $k + 1$, we apply the rule in σ_1 ; the configuration

⁴ this corresponds to the *refractory period* of the neuron in biology

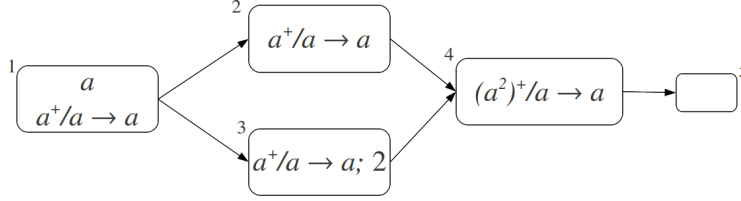


Fig. 1. An example of an SNP system where the source neuron is σ_1 and sink neuron is σ_5 . Neuron σ_4 will only spike once it accumulates two spikes, one each from σ_2 and σ_3 .

would then become $\langle x - 1/2, 0/0, 0/0 \rangle$. At time step $k + 2$, $\langle x - 1/1, 0/0, 0/0 \rangle$. At time step $k + 3$, σ_1 is open again and will now fire one spike to σ_2 , the configuration would now be $\langle x - 1/0, 1/0, 0/0 \rangle$. At time step $k + 4$, σ_1 can again use its rule, and now σ_2 can also apply its rule; the configuration would then become $\langle x - 2/2, 0/0, 1/0 \rangle$.

The initial number of spikes of σ_i is denoted as α_i , $\alpha_i \geq 0$. The initial configuration therefore is $\langle \alpha_1/0, \dots, \alpha_m/0 \rangle$. A *computation* is a sequence of transitions from an initial configuration. A computation may halt (no more rules can be applied for a given configuration) or not.

For SNP systems in this work⁵, we have only one *source* neuron (a neuron without any incoming synapses to it) and one *sink* neuron (a neuron without any outgoing synapses from it). The initial configuration will always be a single spike found only at the source neuron. The objective is to route this spike from the source to the sink neuron. The routes will involve paths in the system, where a *path* consists of at least two neurons σ_i, σ_j such that $(i, j) \in \text{syn}$. Using this idea of a path, we can have four basic routing constructs: (1) sequential, where σ_{i+1} spikes after σ_i and $(i, i+1) \in \text{syn}$, (2) iteration, where at least two neurons spike multiple (possibly an infinite) number of times and a loop is formed e.g. synapses (i, j) and (j, i) exist between neurons σ_i and σ_j , (3) join, where spikes from at least two input neurons σ_m, σ_n are sent to a neuron σ_i , where $(m, i), (n, i) \in \text{syn}$, so that σ_i produces a spike to at least one output neuron σ_j , and $(i, j) \in \text{syn}$, (4) split, where a spike from σ_j is sent to at least two output neurons σ_k and σ_l and $(j, k), (j, l) \in \text{syn}$. An example of an SNP system that we deal with in this work is shown in Fig. 1.

SNP systems considered in this work are those with neurons having exactly one rule only. For SNP systems with delays, notice that if there exists a sequential path from σ_i (with delay $d1$) to σ_j (with delay $d2$) if $d1 < d2$ and σ_i spikes more than once, it is possible for the spikes from σ_i to be lost. This is due to the possibility that σ_j may still be closed when spikes from σ_i arrive. We therefore avoid lost spikes by assuming $d1 \geq d2$ whenever a sequential path from σ_i to σ_j

⁵ more information on SNP systems having input and output neurons as well as non-deterministic SNP systems (we focus on deterministic systems in this work) can be found in [11] and [12] for example.

exists, and leave the cases where $d1 < d2$ as an open problem. Given an SNP system with delays Π and an SNP system without delay $\overline{\Pi}$, by simulation in this work we mean the following: (a) the arrival time t of spikes arrive at sink neurons of Π is the the same time t for the arrival of spikes at the sink neurons of $\overline{\Pi}$ or offset by some constant integer k , (b) the number of spikes that arrive in the sink neurons of Π is the same for the sink neurons in $\overline{\Pi}$ or a factor of the delay of Π . We define the *total runtime* of Π and $\overline{\Pi}$ as the total time needed for a spike to arrive to the sink neurons from the source neurons.

3 Main Results

Now we present our results in simulating SNP systems with delays using SNP systems without delays. Once again we denote SNP systems with delays as Π and those without delays as $\overline{\Pi}$, subject to the restrictions and assumptions mentioned in the previous section.

Definition 1. *Given an SNP system, $\Pi = (O, \sigma_1, \dots, \sigma_m, syn)$, a routing of Π , Π' is defined as*

$$\Pi' = (O, \Sigma', syn'),$$

where:

1. $syn' \subseteq syn$
2. Σ' is a subset of the set of neurons of Π . $\Sigma' = \{\sigma, \hat{\sigma} | (\sigma, \hat{\sigma}) \in syn'\}$.

Lemma 1. *Given an SNP with delay, Π that contains a sequential routing, there exists an SNP without delay, $\overline{\Pi}$ that contains a routing that can simulate the sequential routing of Π .*

Proof. We refer to Fig. 2 for illustrations and designate empty neurons as sink neurons. Let Π be the system in Fig. 2(a). Total runtime for Π is d steps, where final spike count for σ_{12} is $\alpha_{12} = 1$. Now let $\overline{\Pi}$ be Fig. 2(b) that simulates Π . The initial spike count for σ_{21} is $\alpha_{21} = 1 + d$. Total runtime for $\overline{\Pi}$ is $1 + d$ steps, and σ_{22} receives $1 + d$ number of spikes. Final spike count for σ_{23} is $\alpha_{23} = 1$.

Now let Π be the system in Fig. 2(c) and $\overline{\Pi}$ be the system in Fig. 2(d). Π now has two delays, $d1$ and $d2$, and the total runtime for Π is $1 + d1 + d2$. For $\overline{\Pi}$ (with initial spikes $\alpha = (1 + d1)(1 + d2)$), the total runtime is $(1 + d1)(1 + d2) + 3 - 1$. The always open neurons of $\overline{\Pi}$ except for σ_{44} each introduce one time delay i.e. 3, minus one time delay since σ_{44} does not spike. The final spike count for σ_{33} is $\alpha = 1$, which is also the final spike count for σ_{44} . It can be easily shown that addition of neurons without delayed rules in either Π or $\overline{\Pi}$ contribute to a delay of one time step (per neuron) being added to the total runtimes for both systems (see Fig. 3). \square

The total runtime for $\overline{\Pi}$ is one additional time step from the total runtime of Π . The additional time step for $\overline{\Pi}$ comes from delay due to the first spiking of σ_{21} .

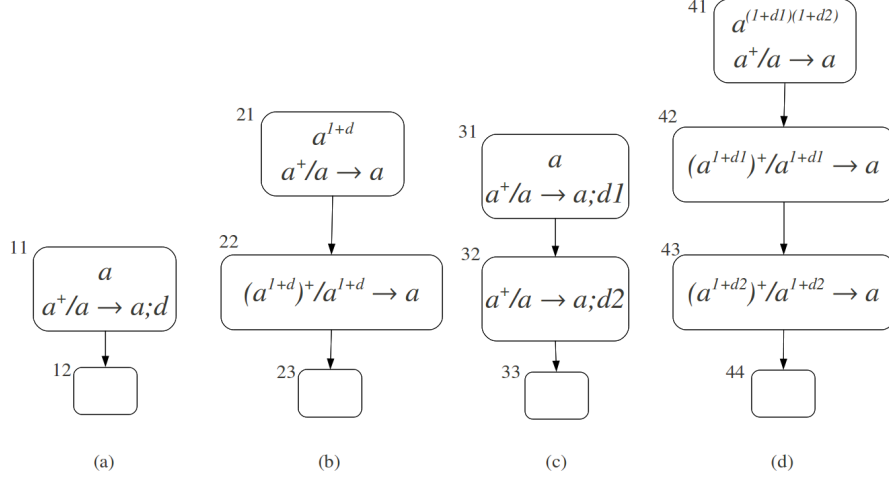


Fig. 2. Sequential routing, with single and multiple delays.

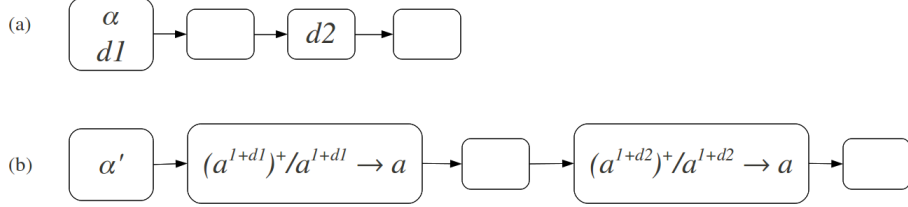


Fig. 3. Sequential routing with additional neurons without delays.

Lemma 2. *Given an SNP with delay, Π that contains an iterative routing, there exists an snp without delay, $\overline{\Pi}$ that contains a routing that can simulate the iterative routing of Π .*

Proof. We refer to Fig. 4 and 5 for illustrations. Let Π be the system in Fig. 4(a) and $\overline{\Pi}$ be the system in Fig. 4(b). Notice that σ_{11} and σ_{12} continuously replenish the spike of one another, forming an infinite loop so that σ_{13} will keep on accumulating spikes. The total runtime is $1 + d$ steps. Neurons σ_{21} and σ_{22} similarly replenish the spike of one another in an infinite loop. The total runtime for $\overline{\Pi}$ is $2 + d$, with one additional delay due to σ_{23} . Due to the operation of σ_{23} , both σ_{13} and σ_{24} obtain one final spike each. It can be easily shown that if the delay of Π is at σ_{11} instead of σ_{12} , the $\overline{\Pi}$ in Fig. 4(b) can still simulate Π .

For iterations with more than one delay, let Π be the system in Fig. 5(a) and $\overline{\Pi}$ be the system in Fig. 5(b). The total runtime of Π is $d1 + d2$ and $(1 + d1)(1 + d2) + 3 - 1$ for $\overline{\Pi}$. The final spike count for Π is $\alpha_{13} = 1$ which is the same as the final spike count for $\overline{\Pi}$ (due to the operation of σ_{24}). \square

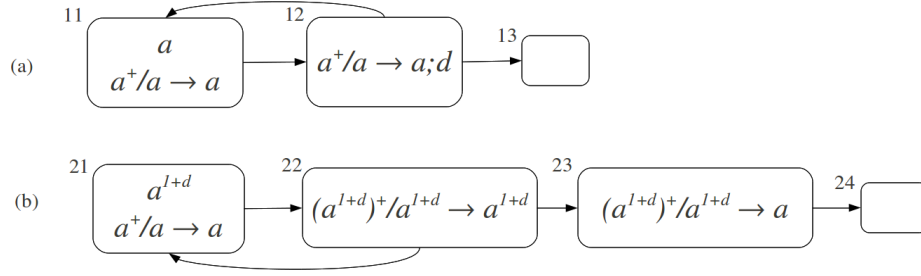


Fig. 4. Iterative routing. Note that the $\overline{\Pi}$ in (b) that simulates Π in (a) is still the same even if σ_{11} has the delay instead of σ_{12}

Notice that the total runtime for $\overline{\Pi}$ for iterative routing with multiple delays is the same as the total runtime for $\overline{\Pi}$ for sequential routing with multiple delays. Again, every additional neuron without delay adds one time delay to the total runtime of Π and $\overline{\Pi}$.

Lemma 3. *Given an SNP with delay, Π that contains a split routing, there exists an SNP without delay, $\overline{\Pi}$ that contains a routing that can simulate the split routing of Π .*

Proof. We refer to Fig. 6 and 7 for illustrations. Let Π be the system in Fig. 6(a) and $\overline{\Pi}$ be the system in Fig. 6(b) and we first consider a split where the neuron σ_{12} that performs the split is the one that has a delay. The total runtime for Π is $1 + d$, with a final spike count of $\alpha_{13} = \alpha_{14} = 1$. The total runtime for $\overline{\Pi}$ is also $1 + d$ with a similar final spike count of $\alpha_{23} = \alpha_{24} = 1$.

Now let Π be the system in Fig. 7(a) and $\overline{\Pi}$ be the system in Fig. 7(b) and we consider a split where the delay is in one of the receiving child neurons (σ_{13}). In this case the sink neurons do not receive a spike at the same time. For Π , if σ_{11} spikes at time t , σ_{14} receives a spike at time $t + 1$ while σ_{15} receives a spike at time $t + 1 + d$. The total runtime therefore (time when both sink neurons obtain one spike) is $1 + d$. For $\overline{\Pi}$ if σ_{21} first spikes at time t then σ_{24} first receives a spike at $t + 1$. The total runtime is $1 + d$ where $\alpha_{25} = 1$ and $\alpha_{24} = 1 + d$. \square

Note that since σ_{24} accumulates $1 + d$ spikes (from σ_{21}), if the split SNP system $\overline{\Pi}$ is a subsystem of a larger SNP system, an additional neuron σ_i with a rule $a^{1+d} \rightarrow a$ needs to be added in order to return the spike number back to one⁶

Lemma 4. *Given an SNP with delay, Π that contains a join routing, there exists an SNP without delay, $\overline{\Pi}$ that contains a routing that can simulate the join routing of Π .*

Proof. We refer to Fig. 8 and 9 for illustrations. Let Π be the system in Fig. 8(a) and $\overline{\Pi}$ be the system in Fig. 8(b) so that we first we consider a join where the

⁶ this additional neuron once again adds one time step of delay.

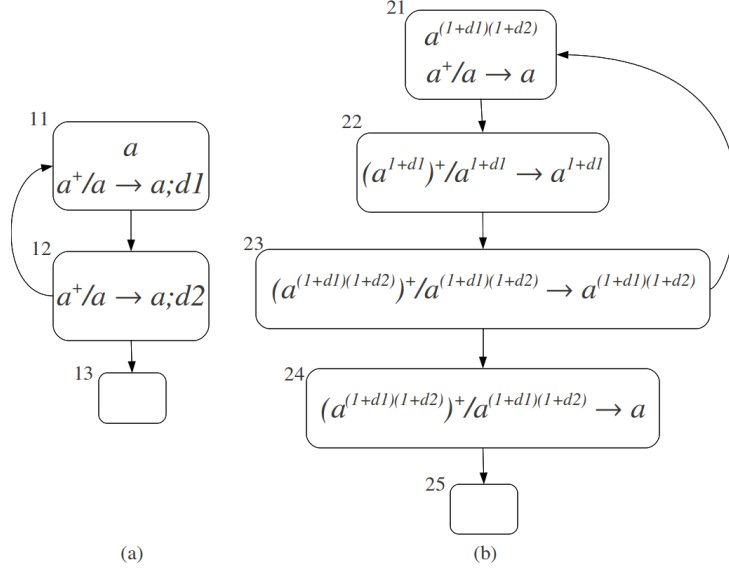


Fig. 5. Iteration with more than one delay.

delay is in a neuron σ_{12} before the neuron σ_{13} that performs the join. The total runtime for Π is $1 + d$ since σ_{13} will wait for the spike from parent neuron σ_{12} to arrive before sending a spike to σ_{14} . The total runtime for $\overline{\Pi}$ is $2 + d$ because of the additional source neuron σ_{21} , since σ_{24} will wait for the spike from σ_{23} . For Π and $\overline{\Pi}$ the final spike count is 1.

Now let Π be the system in Fig. 9(a) and $\overline{\Pi}$ be the system in Fig. 9(b) and we consider next a join where the delay is in the neuron σ_{13} that performs the join. Total runtime for Π is $1 + d$ which is also the total runtime for $\overline{\Pi}$. The final spike count for both systems is 1. \square

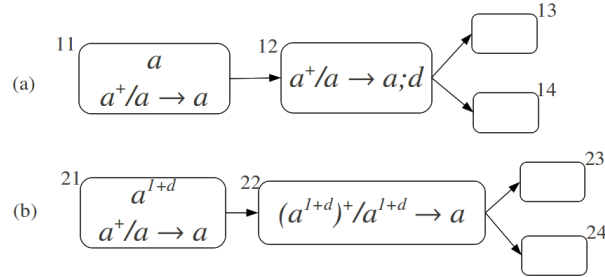


Fig. 6. A split where the neuron performing the split has a delay.

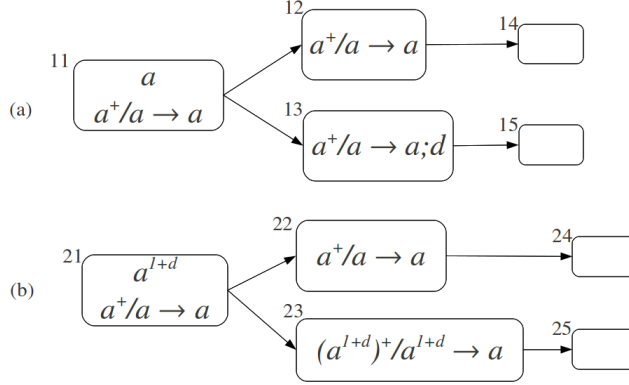


Fig. 7. A split where a child neuron has a delay.

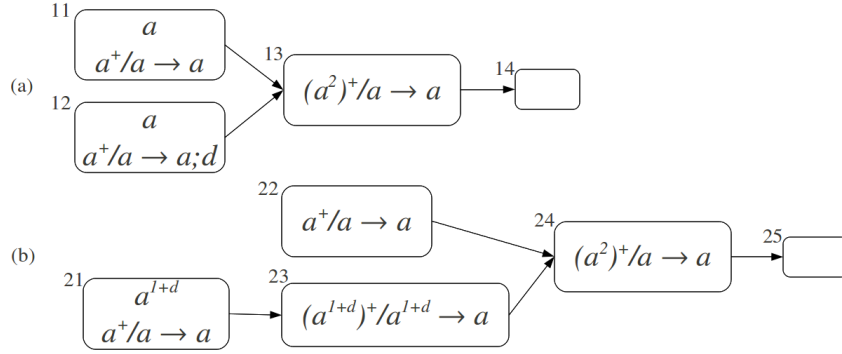


Fig. 8. A join where a parent neuron has a delay.

Theorem 1. *Let Π be an SNP system with delays that follows the restrictions and assumptions given in Section 2 and contains routings limited to the following:*

1. *sequential*
2. *iterative*
3. *split*
4. *join*

Then, there exists an SNP system without delay, $\overline{\Pi}$, that simulates Π .

Proof. Proof follows from Lemma 1, 2, 3, and 4. □

4 Final Remarks

Since $\overline{\Pi}$ has to simulate Π , it is expected that either additional neurons, regular expressions, or spikes are to be added to $\overline{\Pi}$ in order to simulate the delay(s)

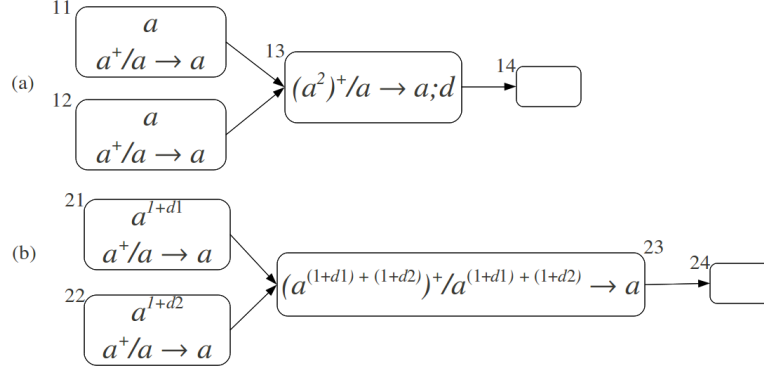


Fig. 9. A join where the neuron performing the join has a delay.

and final configuration of Π (with some offset or spike count multiple of Π). We have shown in this work how to the routing constructs of Π , although we could perhaps do better i.e. less neurons, less initial spikes, and so on.

Conjecture 1. For the case $\alpha = a^k$ where $k > 1$ is an integer, SNP systems without delays can still simulate SNP systems with delays.

Conjecture 2. It is possible to make the time of spiking of SNP systems with delays exactly coincide with the time of spiking of SNP systems without delays (or at least to lessen the neurons or time difference as presented in this work).

We did not consider a split where the delays of the child neurons are not equal, since Lemma 3 only considers a split where only one of the child neurons have a delay (also, it is easy to show that if both child neurons of a split have the same delay d then the $\overline{\Pi}$ in Lemma 3 still holds). As mentioned earlier, we also leave as an open problem on how to simulate SNP systems with delays where $d1 < d2$ for sequential routing.

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